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Laboratory experiments on diapycnal mixing in stratified fluids

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Abstract. Our turbulent lengthscale, velocity and diffusivity scalings are compared with data from other numerical, laboratory and field experiments. Comparison is made with reference to the turbulence intensity measure $\epsilon/\nu N^2$. We showed that our turbulent lengthscale and velocity results are consistent with measurements from the experiments considered, and that the rms turbulent lengthscale L_t is independent of the rate of dissipation of turbulent kinetic energy when $\epsilon/\nu N^2 > 300$. A diffusivity modelled in terms of an advective buoyancy flux, b, is found to reproduce our direct measurements of K_ρ in the experiments considered when $\epsilon/\nu N^2 > 300$. It is shown that modelling K_ρ as $0.2\epsilon/N^2$ is a poor parameterisation of the advective buoyancy flux model in all the experiments considered, and that at large $\epsilon/\nu N^2$, this parameterisation can over-predict the true K_ρ by two orders of magnitude. This overprediction is discussed in terms of a mixing efficiency and it is shown that in the experiments considered the mixing efficiency decreases rapidly with increasing $\epsilon/\nu N^2$. Finally, the application of our diffusivity scaling to other geophysical flows is discussed, and it is shown that a necessary requirement for the use of this scaling is that L_t is independent of the rate of dissipation of turbulent kinetic energy.

Introduction

A clear understanding of the irreversible vertical transport of mass in a stably stratified turbulent flow is fundamental to quantifying the dynamics of density stratified fluids. The rate at which this transport occurs has been been widely modelled as a turbulent diffusivity for mass, K_ρ. Since K_ρ influences the distribution of heat, mass, contaminants and biota throughout a turbulent fluid (e.g. *Tennekes and Lumley*, 1972), an understanding of this quantity is essential to the management of aquatic systems such as lakes, estuaries and the oceans.

The simplest model describing turbulent transport in homogeneous isotropic turbulence relates a turbulent diffusivity K to a turbulent velocity scale U and a turbulent integral lengthscale L (*Tennekes and Lumley*, 1972),

$$K \sim U \times L. \tag{1}$$

This expression for K is considered to represent a bulk eddy diffusivity. *Taylor* (1935) found that the rate of dissipation of turbulent kinetic energy per unit mass, ϵ, is

$$\epsilon \sim \frac{u^3}{l}, \tag{2}$$

where u is the fluctuating component of velocity and l is a linear dimension defining the scale of the turbulent field. If we assume that U, u and L, l are well represented by their root mean square (ms) velocity and length scales U_t and L_t respectively, (1) and (2) can be combined to give

$$K \sim \epsilon^{1/3} L_t^{4/3}. \tag{3}$$

The rate at which a stratifying scalar θ is irreversibly mixed in a turbulent flow has generally been modelled using an average vertical advective flux $\overline{\theta'w'}$, where θ' and w' are the instantaneous scalar and vertical velocity fluctuations respectively (see *Gregg*, 1987, for a full review). This advective flux has been formulated in terms of an eddy coefficient (or diffusivity) K_θ as

$$K_\theta = \frac{\overline{\theta'w'}}{\frac{\partial \overline{\theta}}{\partial z}}, \tag{4}$$

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where $\partial\bar{\theta}/\partial z$ is the mean vertical scalar gradient on which the advective flux acts. *Osborn* (1980) used (4) to model K_ρ in the shear driven environment of the ocean thermocline. By using the turbulent kinetic energy equation, *Osborn* (1980) suggested that $K_\rho \leq 0.2\epsilon/N^2$, where the multiplicative constant is equal to $R_f/(1 - R_f)$, and R_f is a mixing efficiency taken to be less than or equal to 0.15. This model for K_ρ has often been used in geophysical applications other than originally intended, and with the inequality written as an equality such that

$$K_\rho = 0.2\frac{\epsilon}{N^2}. \tag{5}$$

Winters and D'Asaro (1996) argued that rather than modelling K_ρ with (4), it can be computed directly as

$$K_\rho = \frac{\phi_d}{\left|\frac{d\rho}{dz_*}\right|}, \tag{6}$$

where ϕ_d is the irreversible diffusive flux of density across an isoscalar surface in a turbulent flow and $d\rho/dz_*$ is the resorted density gradient. Using this definition, *Barry et al.* (2001) conducted controlled laboratory experiments that directly and independently measured K_ρ, the rate of dissipation of turbulent kinetic energy ϵ, a turbulent lengthscale L_t and the buoyancy frequency $N^2 = -(g/\rho_0)(d\rho/dz)$ in a closed, shear free, linearly salt stratified fluid. Turbulence was generated in the entire fluid volume by the steady motion of a horizontally oscillating vertical rigid grid comprising 1-cm bars at a 5-cm spacing. A schematic diagram of the experimental configuration is shown in Figure 1, and full details are given in *Barry et al.* (2001). *Barry et al.* (2001) described the behaviour of K_ρ with reference to the turbulence intensity parameter $\epsilon/\nu N^2$, where ν is the kinematic viscosity of the fluid. Two regimes were identified: regime E (energetic turbulence, $\epsilon/\nu N^2 > 300$) and regime W (weak turbulence, $\epsilon/\nu N^2 < 300$). It was found that

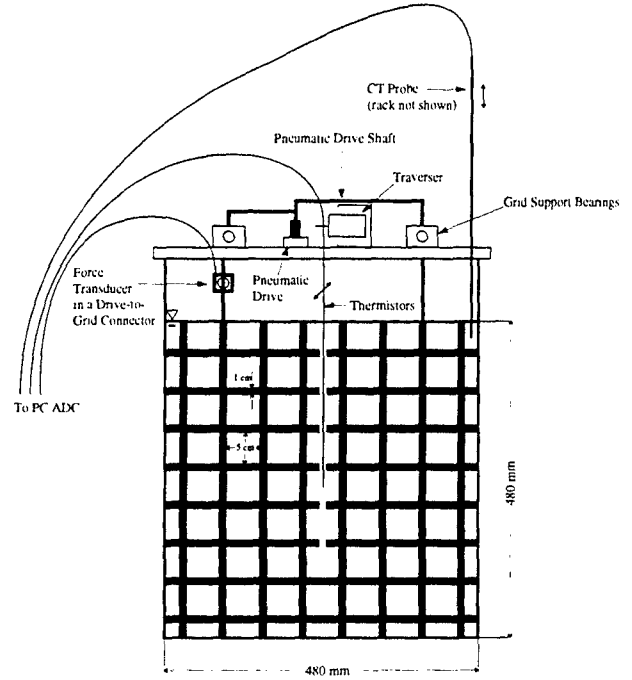$$K_\rho = 24\nu^{2/3}\kappa^{1/3}\left(\frac{\epsilon}{\nu N^2}\right)^{1/3} \text{ and} \tag{7}$$

$$K_\rho = 0.9\nu^{2/3}\kappa^{1/3}\left(\frac{\epsilon}{\nu N^2}\right) \tag{8}$$

for regimes E and W respectively. For regime E, *Barry et al.* (2001) also found that

$$L_t = 20\frac{(\nu\kappa)^{1/4}}{N^{1/2}}, \tag{9}$$

where $(\nu\kappa)^{1/4}/N^{1/2}$ is the convective lengthscale, L_{co}. This lengthscale was related to U_t using (2) (and assuming that l and u are well represented by their respective

Figure 1. Schematic diagram of the front view of the experimental setup. The tank is 520 mm long, and was filled with a linearly salt stratified fluid. The vertical rigid grid stirred the entire fluid volume.

rms quantities) such that

$$U_t = 3.3\left(\frac{\epsilon}{\nu N^2}\right)^{1/3}(\nu N)^{1/2} Pr^{-1/12}, \tag{10}$$

where $Pr = \nu/\kappa$ is the Prandtl number of the fluid. This rms velocity scaling is the same as that obtained by using the direct measurements of K_ρ and L_t and solving for U_t in (1). No velocity or lengthscale relations were identified in regime W.

The primary focus of this paper is to compare the *Barry et al.* (2001) direct diffusivity measurements in regime E with those inferred (via (4)) using the numerical data of *Ivey et al.* (1998), the laboratory data of *Stillinger et al.* (1983b), *Rohr* (1985), and *Itsweire et al.* (1987), and the field data of *Saggio and Imberger* (2001). We also use the same data sets to assess the generality of the *Barry et al.* (2001) lengthscale and velocity relations in the same regime.

The data sets

Direct numerical simulations

Using the code described by *Holt et al.* (1992), *Ivey et al.* (1998) undertook a series of direct numerical simulations (DNS) of stratified shear flows using a 128^3 point computational domain in which N^2 and the square of

the mean vertical shear $S^2 = (\partial U/\partial z)^2$ were held constant. Although only one value of S^2 was used for all the simulations discussed here, N^2 was varied between simulations such that the Richardson number, $Ri = N^2/S^2$, varied from 0.075 to 1.0. The experiments were run for non-dimensional shear time, $0 < St < 8$, however we consider the results from only $St > 2$ where the turbulence becomes self-adjusted and has lost memory of the initialisation (*Ivey et al.*, 1998). At each timestep within a simulation, all turbulent quantities were calculated as ensemble averages over the entire fluid volume. With the application to geophysical flows in mind, we consider only the DNS experiments with $Pr \geq 1$.

Laboratory experiments

The experiments of *Stillinger et al.* (1983b), *Rohr* (1985), and *Itsweire et al.* (1987) measured turbulent quantities at different points downstream of a fixed grid at the head of a water tunnel (see *Stillinger et al.*, 1983a, for a full description of the experimental configuration). These experiments used a variety of grids with bars of diameter 0.318 to 0.635 cm at spacings of 1.905 to 3.81 cm. In all the experiments, salt stratified water was continually recirculated through the tunnel, allowing the collection of timeseries data at several positions downstream of the stationary grid. Average turbulent quantities were computed at these positions by averaging the collected timeseries at each point. The *Rohr* (1985) experiments included mean shear. From numerical simulations, *Itsweire et al.* (1993) found that in these experiments, ϵ was underestimated by approximately a factor of 2. We have applied this correction factor to the data sets for this analysis.

Field experiments

Saggio and Imberger (2001) measured turbulent properties in the metalimnion of Lake Kinneret, Israel, during three consecutive summer periods. During these experiments, a portable flux profiler (PFP) capable of resolving all three turbulent velocity components was traversed vertically through the water column at approximately 10 cm/s. This probe also allowed the measurement of the instantaneous vertical advective buoyancy flux $\rho'w'$, and the rate of dissipation of turbulent kinetic energy, ϵ.

Results

Turbulent lengthscales and velocities

In Figure 2 we plot U_t against $(\epsilon L_t)^{1/3}$, for the above numerical and laboratory data sets where $\epsilon/\nu N^2 > 300$. This rms quantity was computed as
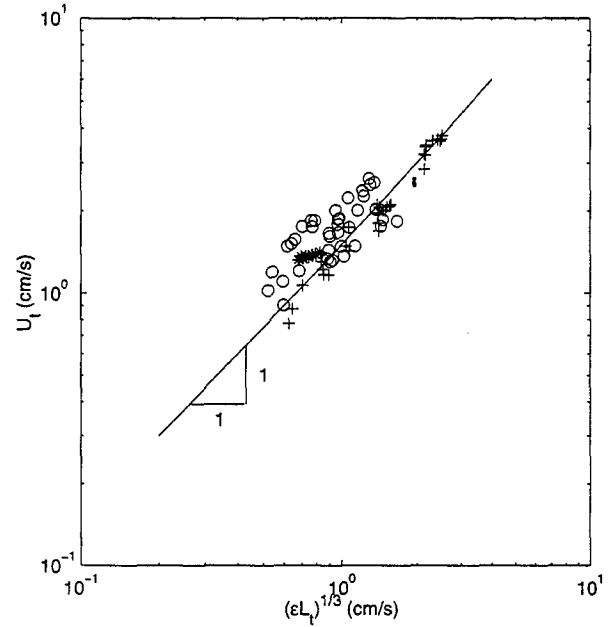
Figure 2. Root mean square turbulent velocity U_t vs $(\epsilon L_t)^{1/3}$, in cm/s; · , *Stillinger et al.* (1983b); ○ , *Rohr* (1985); + , *Itsweire et al.* (1987); * , *Ivey et al.* (1998). The solid line is slope 1, with the equation given by (13).

$$U_t = \sqrt{2\langle u'^2 \rangle + w'^2} \qquad (11)$$

for the laboratory data and

$$U_t = \sqrt{u'^2 + v'^2 + w'^2} \qquad (12)$$

for the DNS data. The solid line in Figure 2 is slope 1 and has

$$U_t = 1.5(\epsilon L_t)^{1/3} \qquad (13)$$

consistent with *Ivey et al.* (1998). Figure 3 shows the normalised turbulent velocity scale against $\epsilon/\nu N^2$ for the same data, where the solid line is given by

$$\frac{U_t}{(\nu N)^{1/2} Pr^{-1/12}} = 4.9 \left(\frac{\epsilon}{\nu N^2} \right)^{1/3}. \qquad (14)$$

Equations (13) and (14) imply that the rms turbulent lengthscale L_t in the DNS and laboratory data is described by

$$L_t = 35 \frac{(\nu\kappa)^{1/4}}{N^{1/2}} = 35 L_{co}. \qquad (15)$$
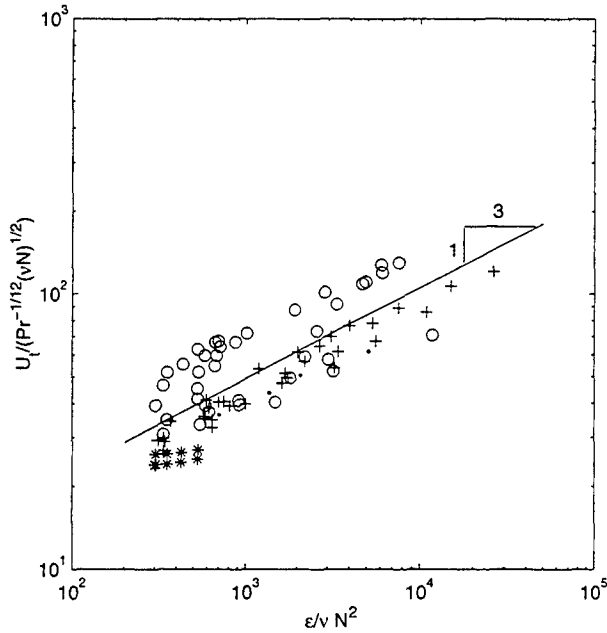
Similarly, *Saggio and Imberger* (2001) found for field data in a heat stratified lake that

$$\frac{U_t}{(\nu N)^{1/2} Pr^{-1/12}} = 3 \left(\frac{\epsilon}{\nu N^2} \right)^{1/3} \qquad (16)$$

and

$$L_t = 15 L_{co}, \qquad (17)$$

where U_t was computed using (12).

Figure 3. Normalised turbulent velocity $U_t/((\nu N)^{1/2}Pr^{-1/12})$ vs $\epsilon/\nu N^2$; · , *Stillinger et al.* (1983b); ○ , *Rohr* (1985); + , *Itsweire et al.* (1987); * , *Ivey et al.* (1998). The solid line is slope 1/3, with the equation given by (14).

Figure 4. $(b/N^2)/(\nu^{2/3}\kappa^{1/3})$ vs $\epsilon/\nu N^2$; · , *Stillinger et al.* (1983b); ○ , *Rohr* (1985); + , *Itsweire et al.* (1987); * , *Ivey et al.* (1998). The equations of the lines are given in the text.

Buoyancy flux estimates of K_ρ

In Figure 4 we plot the quantity $(b/N^2)/(\nu^{2/3}\kappa^{1/3})$ against $\epsilon/\nu N^2$, where $b = (g/\rho_0)\overline{\rho'w'}$ is the buoyancy flux. The solid lines are given by

$$\frac{b}{N^2(\nu^{2/3}\kappa^{1/3})} = 24\left(\frac{\epsilon}{\nu N^2}\right)^{1/3} \qquad (18)$$

for the laboratory data and

$$\frac{b}{N^2(\nu^{2/3}\kappa^{1/3})} = 6\left(\frac{\epsilon}{\nu N^2}\right)^{1/3} \qquad (19)$$
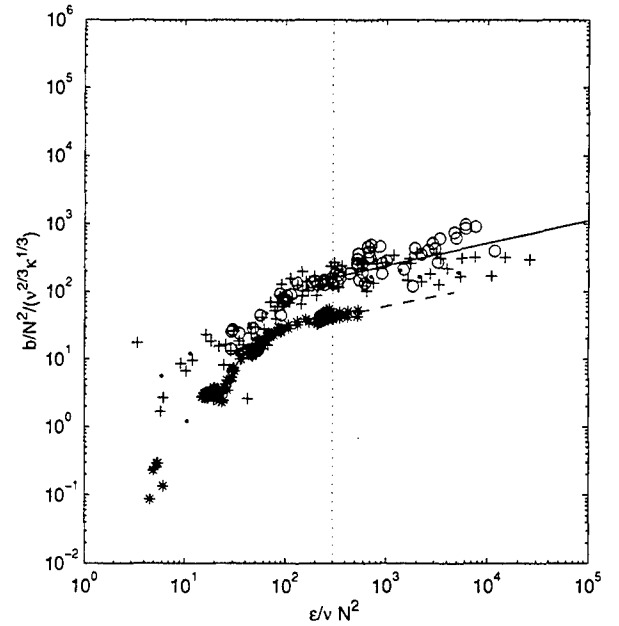
for the DNS data where $\epsilon/\nu N^2 > 300$.

Saggio and Imberger (2001) found from field data in Lake Kinneret that when $\epsilon/\nu N^2 > 36$,

$$\frac{b}{N^2(\nu^{2/3}\kappa^{1/3})} = 12\left(\frac{\epsilon}{\nu N^2}\right)^{1/3}. \qquad (20)$$

Discussion

The functional forms of equations (14), (16) and (15), (17) are consistent with the *Barry et al.* (2001) scalings in (10) and (9), respectively. In particular, (15) and (17) demonstrate that in all these data sets the turbulent lengthscale is not a function of the rate of dissipation of

turbulent kinetic energy when $\epsilon/\nu N^2 > 300$. Further, these velocity scales are consistent with the results of *Barry et al.* (2001) who suggested that for a given N in regime E, an increase in ϵ does not result in an increase in the turbulent overturn scale but, rather, leads to an increase in the magnitude of the rms turbulent velocity.

Although we note some variation in the constant coefficient, the agreement between the turbulent velocity and lengthscale relations of *Barry et al.* (2001) and those of the data sets presented here suggests that, to first order, the configuration of the turbulence generation mechanism does not influence the physics of the turbulence when $\epsilon/\nu N^2 > 300$. Since all data sets are reasonably well described by $L_t \sim L_{co}$ and $U_t/((\nu N)^{1/2}Pr^{-1/12}) \sim (\epsilon/\nu N^2)^{1/3}$, we assume that (1) also applies to the experiments detailed above and, if it were directly measured, K_ρ would be well described by (7) in regime E.

Figure 4 shows that for regime E there is good agreement between (7) (given by the solid line) and $b/(N^2\nu^{2/3}\kappa^{1/3})$ computed using the laboratory data. Although there is some scatter in the laboratory data (consistent with the scatter in the velocity and lengthscale measurements shown in Figures 2 and 3), the trend in $b/(N^2\nu^{2/3}\kappa^{1/3})$ is consistent with (7) in regime E. We conclude that in these laboratory experiments, (4) can be used to satisfactorily approximate the true diffusivity measured by *Barry et al.* (2001) when $\epsilon/\nu N^2 > 300$.
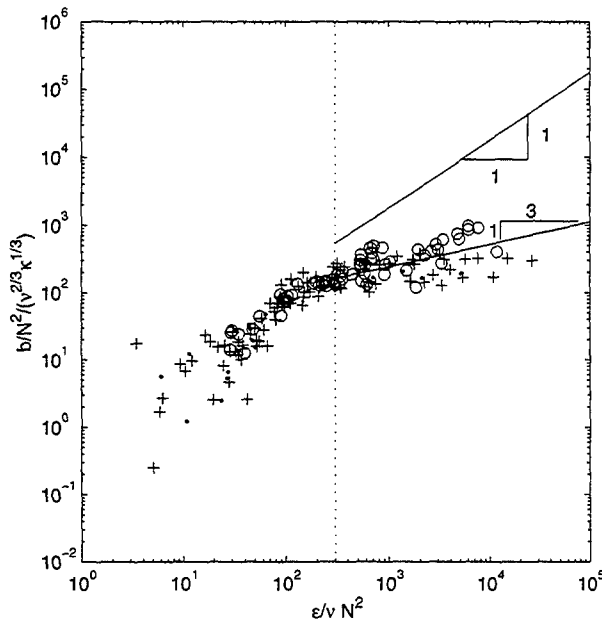
Figure 5. $(b/N^2)/(\nu^{2/3}\kappa^{1/3})$ vs $\epsilon/\nu N^2$; · , *Stillinger et al.* (1983b); ○ , *Rohr* (1985); + , *Itsweire et al.* (1987). The solid line is $(0.2\epsilon/N^2)/(\nu^{2/3}\kappa^{1/3})$, with $Pr = 700$.

The quantity $b/(N^2\nu^{2/3}\kappa^{1/3})$ computed from both DNS and *Saggio and Imberger* (2001) data shows the same functional dependence on $(\epsilon/\nu N^2)^{1/3}$ as (7) when $\epsilon/\nu N^2 > 300$. This implies that the physics of the turbulence in regime E (i.e. the relationship between K_ρ, U_t and L_t) is well described by the scale forms of the *Barry et al.* (2001) results. We note, however, that the constant coefficients in the DNS and field data sets are different from those of the *Barry et al.* (2001) measurements. It is currently not clear why this should be the case.

Figure 5 shows the estimates of K_ρ from the laboratory data sets using (4) compared with the prediction from *Osborn* (1980) in (5) for $Pr = 700$. The figure shows that for $\epsilon/\nu N^2 > 300$, the model predictions from (4) and (5) diverge. For highly energetic turbulence, the two estimates are widely disparate, such that at $\epsilon/\nu N^2 \approx 10^5$ they are different by more than two orders of magnitude.

Barry et al. (2001) argued that the most likely cause of this divergence for large $\epsilon/\nu N^2$, is the selection of a constant mixing efficiency R_f in deriving (5). *Ivey and Imberger* (1991) described the behaviour of this mixing efficiency in terms of the turbulent Froude number,

$$Fr_T = \left(\frac{L_o}{L_C}\right)^{2/3}, \qquad (21)$$

and the turbulent Reynolds number,

$$Re_T = \left(\frac{L_C}{L_k}\right)^{4/3}, \qquad (22)$$

where $L_o = (\epsilon/N^3)^{1/2}$ is the Ozmidov scale, L_C is a centred displacement scale equivalent to L_t, and $L_k = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov scale. In particular, *Ivey and Imberger* (1991) found that R_f has a maximum of approximately 0.2 at $Fr_T \approx 1.2$ (or $L_o \approx L_t$), and that R_f decreases rapidly when Fr_T deviates from this value, such that for $Fr_T > 1.2$,

$$R_f = \frac{1}{1 + 3Fr_T^2} \qquad (23)$$

for large Re_T. If we assume that L_t (and hence L_C) is well described by $27.5L_{co}$ for $\epsilon/\nu N^2 > 300$ (where the coefficient of 27.5 is a representative value from the *Stillinger et al.* (1983b), *Rohr* (1985), *Itsweire et al.* (1987) and *Barry et al.* (2001) data sets), we can substitute $27.5L_{co}$ for L_C in (21) and (22) to give

$$Fr_T = 0.11 \left(\frac{\epsilon}{\nu N^2}Pr^{1/2}\right)^{1/3} \text{ and} \qquad (24)$$

$$Re_T = 83 \left(\frac{\epsilon}{\nu N^2}Pr^{-1}\right)^{1/3}, \qquad (25)$$

respectively. *Ivey and Imberger* (1991) argued that any two of the non-dimensional parameters Fr_T, Re_T and $\epsilon/\nu N^2$ are necessary and sufficient to describe the dynamics of a general turbulent field. However, equations (24) and (25) show that for regime E in the above experiments we need only one.

We plot Fr_T against $\epsilon/\nu N^2 Pr^{1/2}$ in Figure 6, for $\epsilon/\nu N^2 > 300$, where the solid line is given by (24). Equations (23) and (24) show that R_f necessarily decreases with increasing $\epsilon/\nu N^2$,

$$R_f = \frac{1}{1 + 0.04\left(\frac{\epsilon}{\nu N^2}\right)^{2/3}Pr^{1/3}}. \qquad (26)$$

Figure 7 shows R_f against $\epsilon/\nu N^2$ for these data sets, and clearly demonstrates that a diffusivity modelled as $K_\rho = R_f/(1-R_f)\epsilon/N^2$ with R_f constant at 0.15 (shown in Figure 7 as a horizontal solid line) will overestimate the true diffusivity at large $\epsilon/\nu N^2$ in these experiments. This result is consistent with the divergence of the estimates of K_ρ from (4) and (5) shown in Figure 5.

Application to geophysical flows

Equations (24) and (26) hold if, and only if, the turbulent lengthscale L_t is well described by L_{co}. Several field investigations however, (e.g. *Crawford*, 1986; *Gregg*, 1987; *Peters et al.*, 1988; *Imberger and Ivey*,
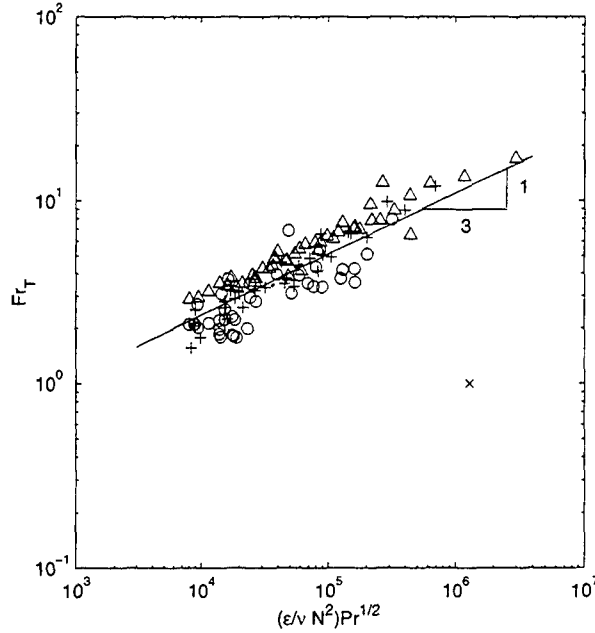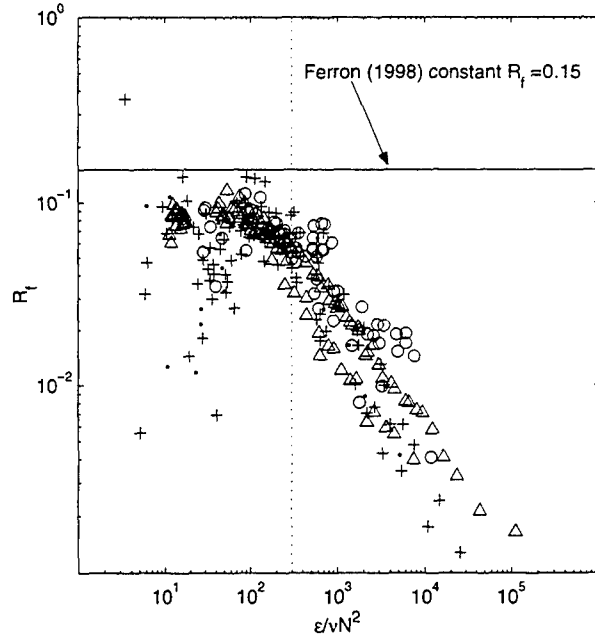
1991; *Peters et al.*, 1995; *Ferron et al.*, 1998) have found that in some geophysical flows $L_t \sim L_o$, or equivalently, that $Fr_T \approx 1$. In these situations, this relationship between L_t and L_o has been found to hold regardless of the value of $\epsilon/\nu N^2$. For instance, *Ferron et al.* (1998) observed that in the Romanche fracture zone,

$$L_o = 0.95(\pm 0.6)L_T, \tag{27}$$

even when $\epsilon/\nu N^2 \approx 500,000$, where L_T is the Thorpe displacement scale and is a measure of L_t. The corresponding value of Fr_T is order 1 and is shown in Figure 6 as an $\times$. The *Ferron et al.* (1998) measurements clearly do not conform to the trends of the data sets previously considered here, fundamentally because L_t does not scale like L_{co}.

The measurements of *Ferron et al.* (1998) imply that Fr_T is *always* approximately unity, regardless of the magnitude of $\epsilon/\nu N^2$. The arguments of *Ivey and Imberger* (1991) then require that the mixing efficiency in the Romanche fracture zone is constant and maximal, and hence the large value of $\epsilon/\nu N^2$ cannot be used in isolation to infer that R_f is vanishingly small. Rather, either Fr_T or Re_T must also be included in a description of the turbulent dynamics. In such cases, the scaling results of *Barry et al.* (2001) should not be expected to hold.

Conclusions

We have compared the turbulent lengthscale, velocity and diffusivity scalings presented by *Barry et al.* (2001) with data from *Ivey et al.* (1998) DNS experiments, laboratory experiments of *Stillinger et al.* (1983b), *Rohr* (1985), and *Itsweire et al.* (1987) and the field experiments of *Saggio and Imberger* (2001). For $\epsilon/\nu N^2 > 300$, the *Barry et al.* (2001) velocity and lengthscale results are good descriptors of the physics of the turbulence in all these experiments, to constant coefficient. We have shown that the turbulent lengthscales in all these experiments in this regime are independent of the rate of dissipation of turbulent kinetic energy. This in turn suggests that for a given N, an increase in ϵ does not increase the turbulent lengthscale, but rather, leads to an increase in the magnitude of the turbulent velocity.

We have shown that when $\epsilon/\nu N^2 > 300$, a diffusivity modelled using (4), is a good representation of the direct measurements of *Barry et al.* (2001) for the experiments considered, to constant coefficient.

We have discussed the parameterisation of (4) as $K_\rho = 0.2\epsilon/N^2$ and have shown that, in the data sets considered, this overestimates the true diffusivity for $\epsilon/\nu N^2 > 300$. We have suggested that this is a consequence of selecting a constant mixing efficiency $R_f =$

Figure 6. Turbulent Froude number Fr_T vs $(\epsilon/\nu N^2 Pr^{1/2})^{1/3}$; $\cdot$, *Stillinger et al.* (1983b); $\circ$, *Rohr* (1985); $+$, *Itsweire et al.* (1987); $\triangle$, *Barry et al.* (2001); $\times$, *Ferron et al.* (1998). The solid line is $0.11(\epsilon/\nu N^2 Pr^{1/2})^{1/3}$.

Figure 7. R_f vs $\epsilon/\nu N^2$; $\cdot$, *Stillinger et al.* (1983b); $\circ$, *Rohr* (1985); $+$, *Itsweire et al.* (1987); $\triangle$, *Barry et al.* (2001).

0.15, as for increasing $\epsilon/\nu N^2$, R_f necessarily decreases in the data sets considered.

Finally, we have discussed the application of the *Barry et al.* (2001) scalings to geophysical flows and have suggested that a necessary requirement for the use of these scalings is that the turbulent lengthscale L_t is independent of ϵ. If this constraint is satisfied (i.e. $L_t \sim L_{co}$) then K_ρ, and indeed all the turbulent dynamics, can be described in terms of only a single parameter $\epsilon/\nu N^2$. When this lengthscale constraint is not satisfied, the *Barry et al.* (2001) scalings do not apply. Future work must now investigate the circumstances under which this lengthscale constraint is satisfied in a variety of geophysical flows.

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